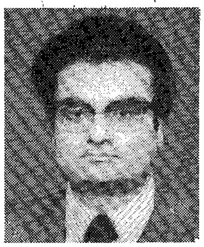

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Short Papers

Determination of the Characteristic Impedance by a Step Current Density Approximation

STEPHAN A. IVANOV AND GEORGI L. DJANKOV

Abstract —**The step current densities are used to determine the characteristic impedance of a transmission line with rectangular shape of the conductors. Numerical results for different rectangular lines with asymmetrical position of the inner conductor are presented. The comparison of the results for the square and rectangular coaxial lines shows quite good agreement with the known data.**

I. Introduction

The characteristic impedance of the rectangular coaxial transmission line can be determined with good accuracy for all cases of interest [1]. When the axis of the inner and outer conductors does not coincide and their dimensions differ considerably, the problem becomes complicated. A general expression for the characteristic impedance is derived in [2], but no numerical data are given for the line with asymmetrical position of the conductors. Also, a doubly eccentric rectangular line is considered by Chen [3] for the case of a sufficiently small gap between conductors. An analytical expression for the characteristic impedance of the rectangular line with arbitrary dimensions can be found in [4]. Since the impedance in [4] is calculated by the utilization of the mean value current densities, which may differ considerably from the true current distribution on the inner conductor surface, an error of several percent exists. The purpose of the present short paper is to improve the accuracy of the characteristic impedance calculation by using the step current density approximation. In this way, the edge discontinuity of the current distribution is taken into consideration and an error less than one percent for the impedance values can be obtained.

II. Theoretical Results

The investigated rectangular transmission line is shown in Fig. 1. Since the cross section of this line coincides with the single cell of the four-conductor line considered in [4], the characteristic impedance can be determined by the expression (4) in [4]—the case of the odd–odd mode of excitation. Here it is proposed that the current distribution $J_k(1)$ be replaced by a set of step current densities J_{kq}. For the case when the l_k-wall is divided into N_k intervals with a length l_k/N_k, the final result for the characteristic impedance of the investigated line can be expressed by the formula

$$Z\sqrt{\frac{\epsilon_r}{\mu r}}=\frac{120}{\pi^2}\frac{\sum\limits_{i,j=1}^{4}\sum\limits_{q,r=1}^{N_k}J_{iq}J_{jr}\sum\limits_{m=1}^{\infty}\frac{\beta_{irm}\beta_{jrm}Z_{ijqrm}}{m^3(1-e^{-2m\pi B})}}{\left[\sum\limits_{q=1}^{N_{1,2}}(J_{1q}+J_{2q})\frac{W}{N_{1,2}}+\sum\limits_{q=1}^{N_{3,4}}(J_{3q}+J_{4q})\frac{T}{N_{3,4}}\right]^2}$$

where

$$\beta_{1rm}=\sin m\pi D \quad \beta_{2rm}=\sin m\pi(D+T)$$

$$\beta_{3rm}=\beta_{4rm}=\cos m\pi\left(D+T\frac{r-1}{N_{3,4}}\right)-\cos m\pi\left(D+T\frac{r}{N_{3,4}}\right).$$

The coefficients $Z_{ijqrm}=Z_{jiqrm}$ are determined as follows:

$$Z_{11qrm}=Z_{12qrm}=Z_{22qrm}=e^{-m\pi W(q-r+1/N_{1,2})}\cdot\left[1+e^{-2m\pi(B-W(q-r+1/N_{1,2}))}\right]$$
$$+e^{-m\pi W(q-r-1/N_{1,2})}\left[1+e^{-2m\pi(B-W(q-r-1/N_{1,2}))}\right]$$
$$-2e^{-m\pi W(q-r/N_{1,2})}\left[1+e^{-2m\pi(B-W(q-r/N_{1,2}))}\right]$$
$$-\left(1-e^{-m\pi(W/N_{1,2})}\right)^2\left[e^{-m\pi(2S+W(q+r-2/N_{1,2}))}+e^{-m\pi(2B-2S-W(q+r/N_{1,2}))}\right]$$

$$Z_{13qrm}=Z_{23qrm}=\left(1-e^{-(m\pi W/N_{1,2})}\right)\left(1-e^{-2m\pi S}\right)\cdot\left[e^{-m\pi W(q-1/N_{1,2})}+e^{-m\pi(2B-2S-W(q/N_{1,2}))}\right]$$

$$Z_{33qrm}=\left(1-e^{-2m\pi S}\right)\left[1-e^{-2m\pi(B-S-W)}\right]$$

$$Z_{14qrm}=Z_{24qrm}=\left(1-e^{-(m\pi W/N_{1,2})}\right)\cdot\left[1-e^{-m\pi(2S+W(2q-1/N_{1,2}))}\right]\cdot\left[e^{-m\pi W(1-(q/N_{1,2}))}-e^{-m\pi(2B-2S-W(1-(q/N_{1,2})))}\right]$$

$$Z_{34qrm}=e^{-m\pi W}\left(1-e^{-2m\pi S}\right)\left[1-e^{-2m\pi(B-S-W)}\right]$$

$$Z_{44qrm}=\left[1-e^{-2m\pi(S+W)}\right]\left[1-e^{-2m\pi(B-S-W)}\right].$$

Manuscript received March 1, 1983; revised November 14, 1983.

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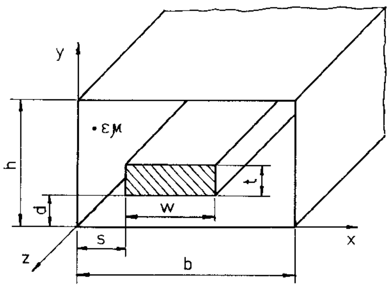

Fig. 1. Rectangular transmission line.

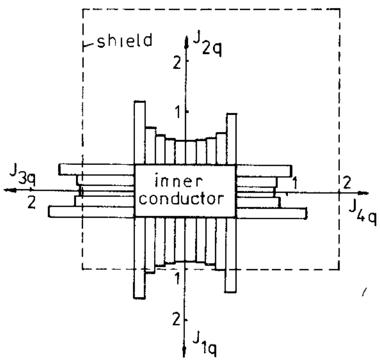

Fig. 2. Step current densities distribution for the line with dimensions: $B = 1$, $S = D = 0.2$, $W = 0.4$, $T = 0.2$.

TABLE I
CHARACTERISTIC IMPEDANCE VALUES OF THE SQUARE COAXIAL LINE WITH DIMENSIONS $B = 1, 2S + W = B, 2D + T = 1$

W=T	0.1	0.2	0.3	0.4	0.5	0.6
$Z_{ref.1}$	132.65	91.11	66.87	49.82	36.81	26.46
Z_{step}	132.38	90.82	66.87	49.53	36.97	26.40

All dimensions are normalized with respect to the height of the shield, i.e., $B = b/h$, $D = d/h$, $S = s/h$, $W = w/h$, and $T = t/h$.

The step current densities J_{kq} can be determined in different ways: analytically, by electrolytic tank measurements, or by other appropriate modeling. Below, the step current densities J_{kq} are calculated by the self-consistent field method (SCFM), described in [5], where the recurrent expression for the case under consideration is obtained. As an illustration of the SCFM, the step current density distribution for the line with dimensions $B = 1$, $S = D = 0.2$, $W = 0.4$, and $T = 0.2$ is shown in Fig. 2.

III. NUMERICAL RESULTS

The calculation of the characteristic impedance is made by a computer. First, the algorithm is checked for the square coaxial line. The number of steps N_k is chosen to be 10 for all walls and the step current densities are calculated with three iterations [5]. A comparison of our results with Bowman's data [1] is shown in Table I. For all cases, the difference is quite small—less than 0.5 percent. More detailed investigation (see Table II) shows that, for the square coaxial line, it is enough to use about $3 \sim 5$ steps and $2 \sim 3$ iterations.

Further, several cases are taken into consideration.

1) The inner conductor is situated symmetrically in the square shield—$B = 1$, $2S + W = B$, $2D + T = 1$. The numerical data for this case are presented in Table III, where a comparison with the results obtained by utilization of the mean value current densities [4] is done. One can conclude that the step current approximation improves the accuracy of the characteristic impedance calculation by $1 \sim 2$ percent.

2) The inner conductor is situated close to the bottom of the square shield—$B = 1$, $2S + W = B$, $D = 0.1$. The corresponding

TABLE II
DEPENDANCE OF THE CHARACTERISTIC IMPEDANCE Z_{step} (OHMS) WITH THE NUMBER OF ITERATIONS p AND NUMBER OF STEPS N_k FOR THE SQUARE COAXIAL LINE WITH DIMENSIONS $B = 1$, $2S + W = B, 2D + T = 1$ AND $W = T = 0.2$

p \ N_k	10	7	5	3	1
2	90.8837	90.9441	91.0318	91.2573	91.5564
3	90.8223	90.8935	90.9952	91.2394	91.5449
4	90.8164	90.8887	90.9931	91.2417	91.5500

TABLE III
CHARACTERISTIC IMPEDANCE VALUES OF THE RECTANGULAR COAXIAL LINE WITH DIMENSIONS $B = 1, 2S + W = B, 2D + T = 1$

T	W	Z_{step}	Z_{mean}	ΔZ/Z (%)
0.05	0.2	121.63	123.17	1.3
0.1	0.2	108.53	110.66	1.9
0.1	0.4	80.32	81.35	1.2
0.2	0.2	91.20	92.99	1.9
0.2	0.4	67.20	68.98	2.6
0.2	0.6	50.02	51.88	3.5

TABLE IV
CHARACTERISTIC IMPEDANCE VALUES OF THE RECTANGULAR LINE WITH DIMENSIONS $B = 1, 2S + W = B, D = 0.1$

T	0.1		0.2	
W	0.2	0.4	0.2	0.4
Z_{step}	69.54	47.86	62.04	43.27
Z_{mean}	71.41	49.14	64.97	44.98
ΔZ/Z (%)	2.7	2.7	4.7	3.9

TABLE V
CHARACTERISTIC IMPEDANCE VALUES OF THE ASYMMETRICAL RECTANGULAR LINE WITH DIMENSIONS $B = 1, S = D = 0.2$

T	W	Z_{step}	Z_{mean}	ΔZ/Z (%)
0.05	0.2	97.20	98.09	1.0
0.1	0.2	86.78	88.54	2.0
0.1	0.4	65.58	66.41	1.2
0.2	0.2	73.54	75.63	3.7
0.2	0.4	56.73	58.41	2.9
0.2	0.6	44.04	45.30	2.8

data are presented in Table IV. In this case, the difference between Z_{step} and Z_{mean} increases, especially for the thicker conductors.

3) The inner conductor is situated asymmetrically in the square shield—$B=1$, $S=D=0.2$. As follows from Table V, the effectiveness of the proposed method is greater for the thicker conductors.

IV. Conclusion

The numerical data presented in Tables I–V show that the utilization of the SCFM with the step current density approximation makes it possible to calculate the characteristic impedance of a different TEM transmission line with rectangular shape of the conductors with good accuracy. For the lines with symmetry, even the one step approximation is enough, while the general case of an asymmetrical position of the thick inner conductor into the shield needs the utilization of the step current densities.

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Definition of Nonlinear Reflection Coefficient of a Microwave Device Using Describing Function Formalism

J. OBREGON AND F. FARZANEH

***Abstract*—At microwaves, it is necessary to define rigorously the large signal reflection coefficient of a nonlinear device. In this paper, the describing function concept is applied to the power waves incident on, and reflected by, a nonlinear element.**

This method allows us to define the nonlinear reflection coefficient (NLRC) on the power wave basis.

This NRLC is then compared with that defined on the current or voltage basis.

Numerical calculations applied to nonlinear elements illustrate the theoretical results.

I. Introduction

To use nonlinear devices, one might generalize linear concepts such as impedance, admittance, and transfer function by the so-called describing function method [1], [2]. These quantities would be defined for given input signals.

At microwaves, generally, the quantity measured is the reflection coefficient (or S-parameters for n-port devices). So it is necessary to define exactly the nonlinear reflection coefficient concept and its relation with the nonlinear impedance.

Manuscript received July 6, 1983; revised October 21, 1983.
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In this short paper, we define the nonlinear reflection coefficient by application of the describing function (DF) method to the power waves, then we compare the nonlinear reflection coefficient (NLRC) and the nonlinear impedance of the same device. Some numerical results concerning nonlinear elements will be given.

At microwave frequencies, what is measured is the power of reflected or incident waves; thus, one obtains by direct measurement the reflection coefficient (S-parameters) [5]. So we should define the describing function in terms of incident and reflected waves. The definitions of the incident and reflected waves at the device terminals are

$$a(t)=\frac{v(t)+Z_0 i(t)}{2\sqrt{Z_0}} \tag{1}$$

$$b(t)=\frac{v(t)-Z_0 i(t)}{2\sqrt{Z_0}} \tag{2}$$

where Z_0 is the reference impedance, and i and v are instantaneous current and voltage in the device.

In a linear circuit, the reflection coefficient is defined as

$$\Gamma(\omega)=\frac{\mathrm{F}\{b(t)\}}{\mathrm{F}\{a(t)\}} \tag{3}$$

where F stands for Fourier Transform which can be also expressed as

$$\Gamma(\omega)=\frac{Z(\omega)-Z_0}{Z(\omega)+Z_0}=\frac{Y_0-Y(\omega)}{Y_0+Y(\omega)} \tag{4}$$

but this does not hold for the nonlinear case.

II. Nonlinear Resistance

Let us suppose that the instantaneous relation between i and v (for a purely resistive device) is $i=f(v)$.

Substituting into [1] and [2], we obtain

$$a(t)=\frac{v+Z_0 f(v)}{2\sqrt{Z_0}}$$
$$b(t)=\frac{v-Z_0 f(v)}{2\sqrt{Z_0}}. \tag{5}$$

b and a are parametrically related; one can then draw the characteristic curve and obtain

$$b=g(a). \tag{6}$$

By application of the describing function method, one can seek a linear approximation of this relation by putting

$$\int_0^T \frac{\partial}{\partial \Gamma_{\text{NL}}}\{g(a)-\Gamma_{\text{NL}}\cdot a)\}^2\,dt=0. \tag{7}$$

Γ_{NL} is immediately deduced as

$$\Gamma_{\text{NL}}=\frac{\int_0^T a g(a)\,dt}{\int_0^T a^2\,dt} \tag{8}$$

and one can write

$$B=\Gamma_{\text{NL}}A$$

where B and A are amplitudes of the reflected and incident waves.

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